identical transformation $A \to A$. Then $\bar{U}$ may be chosen of type Δ, and the number $(\Gamma \cdot \Delta)$ obtained is precisely (24).

Let us recall in concluding that the same formulas hold for transformations of compact metric HLC spaces. They are spaces endowed with a strong type of local connectedness in the sense of homology, analogous to that possessed by the so-called absolute neighborhood retracts.†

PRINCETON UNIVERSITY

CIRCLES IN WHICH $|F(x)|$ HAS A SINGULARITY OR ASSUMES PREASSIGNED VALUES

BY J. W. CELL

Let k be a given positive integer and let a_0 and $a_k \neq 0$ be two given constants. Let $F_k(x)$ be any member whatever of the class C_k of functions which are regular in the neighborhood of the origin and which there have the expansion

$$F_k(x) = a_0 + a_k x^k + a_{k+1} x^{k+1} + \cdots,$$

where a_0 and a_k are the two given constants.

THEOREM 1. *Let $\eta(a_0, a_1) = 0$ if $|a_0| = 1$. In case $|a_0| < 1$, let $\eta(a_0, a_1) = \{1 - |a_0|^2\}/|a_1|$, and if $|a_0| > 1$, let $\eta(a_0, a_1) = \{2|a_0| \log |a_0|\}/|a_1|$. Then in or on the circle $|x| = \eta(a_0, a_1)$, either $F_1(x)$ has a singularity or $|F_1(x)|$ assumes the value one. Moreover, no smaller radius will do for the whole class of functions C_1.*

COROLLARY. $\eta(a_0, 1) = |a_1| \; \eta(a_0, a_1)$.

PROOF. If $|a_0| = 1$, the theorem is granted, so we shall henceforth suppose that this is not the case. If $a_0 = re^{i\alpha}$, $(r \geqq 0)$, we define $E(x) = e^{-i\alpha} F_1(x)$. Then $|E(x)| = |F_1(x)|$ and hence we may, with no loss of generality in the proof, suppose that a_0 is real and non-negative.

CASE 1. $0 \leqq a_0 < 1$. There exists a positive number η such that for $|x| \leqq \eta$, $F_1(x)$ is regular and $|F_1(x)| < 1$. Now form

$$G(x) = \frac{F_1(x) - a_0}{-a_0 F_1(x) + 1}. \tag{1}$$

† See Duke Mathematical Journal, vol. 2 (1936), pp. 435–442.