

POSTULATES FOR BOOLEAN ALGEBRA IN
TERMS OF TERNARY REJECTION*

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1. *Introduction.* The operation of ternary rejection‡ in Boolean algebra is the operation $()$ given by $(abc) = a'b' + b'c' + c'a'$. In this paper, I shall present a set of postulates for Boolean algebras in which ternary rejection is taken as the only primitive idea, besides that of *class*. As a result, all the special Boolean elements are introduced with an elegance not possible in any other set known to the author. Thus, the *negative* of an element is defined in terms of the primitive ideas, and then *any* two contradictory elements are chosen to represent the *zero* element and the *universe* element of the resulting Boolean algebra.

We prove the *sufficiency* of the new postulates for Boolean algebra by deriving from them the well known Whitehead-Huntington set;§ the proof of *necessariness* consists in the converse derivation. Finally, we establish the *consistency* and *independence* of the postulates by furnishing proof-systems of the usual type.

2. *The New Postulates.* The new postulates have as undefined ideas a *class* K and a *ternary operation* $()$. The postulates are the propositions A_1 - A_5 below. In Postulates A_3 - A_5 the condition *if the elements involved and their indicated combinations belong to K* is to be understood.

POSTULATE A_1 . K contains at least two distinct elements.

POSTULATE A_2 . If a, b, c are elements of K , (abc) is an element of K .

* Presented to the Society, December 31, 1936.

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‡ For other papers dealing with ternary operations see A. B. Kempe, *On the relation between the logical theory of classes and the geometrical theory of points*, Proceedings of the London Mathematical Society, (1), vol. 21 (1890), pp. 147-182; Orrin Frink, *The operations of Boolean algebras*, Annals of Mathematics, (2), vol. 27 (1925-1926), pp. 477-490; see also the bibliography at the end of Frink's paper.

§ See the Transactions of this Society, vol. 5 (1904), pp. 288-309.