the integrals taken so that the boundary C is traversed in the positive sense. Series (15) *converges uniformly for x in every closed region in* $\mathcal{L}$, *and is therefore valid for all x in* $\mathcal{L}$.

From this we obtain the following result.

THEOREM 2.* *The series*

$$y(x) = \sum_{n=0}^{\infty} f_n \frac{x^n}{n!} \tag{17}$$

converges in a circle of radius exceeding $1/2$, *and in some neighborhood of* $x = -1/2$ *the function* $y(x)$ *is a solution of the equation*

$$\Delta y(x) = F(x). \tag{1}$$

PENNSYLVANIA STATE COLLEGE

ON THE SUMMABILITY BY POSITIVE TYPICAL MEANS OF SEQUENCES $\{f(n\theta)\}$†

BY M. S. ROBERTSON

1. *Introduction.* In a recent paper‡ the author required an inequality for the expression

$$\lim_{n\to\infty} \frac{1}{n} \sum_{k=1}^{n} |\sin k\theta| \leqq \frac{1}{\pi}\int_0^{\pi} |\sin\theta|\, d\theta = \frac{2}{\pi}, \tag{1}$$

which apparently is due to T. Gronwall.§ This inequality suggests immediately the question: For what functions $f(\theta)$, defined in the interval $(-\pi, \pi)$, are we permitted to write

$$F(\theta; f) \equiv \lim_{n\to\infty} \frac{1}{n} \sum_{k=1}^{n} f(k\theta) \leqq \frac{1}{2\pi}\int_{-\pi}^{\pi} f(\theta)d\theta? \tag{2}$$

More generally, we may ask: For what functions $f(\theta)$ and sequences $\{a_n\}$ of positive numbers is the following true:

---

* See Transactions of this Society, loc. cit., p. 359.

† Presented to the Society, April 11, 1936.

‡ See M. S. Robertson, *On the coefficients of a typically-real function*, this Bulletin, vol. 41 (1935), p. 569.

§ See Transactions of this Society, vol. 13 (1912), pp. 445–468.