ON CERTAIN CONFIGURATIONS OF POINTS IN SPACE AND LINEAR SYSTEMS OF SURFACES WITH THESE AS BASE POINTS*

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1. *Introduction.* Configurations of this sort in connection with certain surfaces are known in large numbers. For example, the vertices of the 45 triangles formed by the 27 lines on a general cubic surface; the 12 vertices of 3 desmic tetrahedra; the 24 double points of the 6 quintic cycles of the symmetric collineation group on five variables interpreted in S_3; the G_{18} group of points which I found on a new normal form of the cubic surface,† and so on.

In this paper I shall establish two new configurations of points and investigate their properties and some of the surfaces on these points.

2. *The G_{27} of W-Points.* This configuration is defined by the system of points W

$$(1)\quad W = (\omega^\alpha, \omega^\beta, \omega^\gamma, 1), \quad \omega^3 = 1, \quad \alpha, \beta, \gamma \equiv 0, 1, 2, (\mathrm{mod}\, 3),$$

which yields the group G_{27} of 27 points W. Consider now any of the W's and two more of the set as follows:

$$\begin{aligned} W_0 &= (\omega^\alpha, \quad \omega^\beta, \quad \omega^\gamma, \quad 1), \\ W_1 &= (\omega^{\alpha+1}, \omega^{\beta+1}, \omega^{\gamma+1}, 1), \\ W_2 &= (\omega^{\alpha+2}, \omega^{\beta+2}, \omega^{\gamma+2}, 1). \end{aligned}$$

Subtracting corresponding coordinates of these three points, say $(W_0 - W_1)$, $(W_1 - W_2)$, $(W_2 - W_0)$, and dividing in each case by $(1-\omega)$, we obtain the point $V(\omega^\alpha, \omega^\beta, \omega^\gamma, 0)$. The cross-ratio of the four points is

$$(VW_0W_1W_2) = (\infty, \omega^\alpha, \omega^{\alpha+1}, \omega^{\alpha+2}) = (\infty, 1, \omega, \omega^2) = -\omega^2.$$

THEOREM 1. *Every V-point is collinear with three W-points. The cross-ratio of these four points is equianharmonic.*

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† American Journal of Mathematics, vol. 53 (1931), pp. 902–910.