# A GENERALIZED ELEMENT OF DECOMPOSITION FOR DOUBLY PERIODIC FUNCTIONS*

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1. *Introduction.* Hermite† has defined a function $f(x)$ to be doubly periodic of the third kind if it is meromorphic and satisfies two periodicity relations of the form

$$f(x + 2K) = e^{\alpha x+\beta}f(x),$$
$$f(x + 2iK') = e^{\gamma x+\delta}f(x).$$

These relations may be transformed into the following ones

$$(1)\qquad \begin{aligned} f(x + \pi) &= f(x), \\ f(x + \pi\tau) &= e^{2i(ax+b)}f(x), \end{aligned}$$

and it has been shown that the constant $a$ in (1) is an integer and is equal to the excess of the number of poles over the number of zeros of $f(x)$ in a primitive period cell. For our purposes in the following it is helpful to consider elliptic functions and doubly periodic functions of the second kind as special cases of the above, namely, when $a=b=0$, and $a=0$, respectively.

We derive a generalized element of decomposition which by proper specialization of the constants can be used equally well to obtain expansions for doubly periodic functions of the first, second, or third kinds. It should be noted, however, that the method of the present paper fails to yield a decomposition in the case of doubly periodic functions of the third kind having more zeros than poles, that is, when $a<0$, since the convergence of the series representing the integrals for $C_r$ in the following is then destroyed. This same difficulty has been met by other writers in discussing the decomposition of doubly periodic functions of the third kind.‡ However, Appell has solved this difficulty by interchanging the roles of $x$ and $y$ in his element of decomposition obtained for the case where $a$ is positive.

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† Hermite, Oeuvres, vol. 3, p. 329 (footnote); Appell, Mémorial des Sciences Mathématiques, fascicule 36 (1929).

‡ See, for example, a paper by Basoco in Acta Mathematica, vol. 57.