A NOTE ON MATRICES DEFINING TOTAL REAL FIELDS*

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Let K be algebraic of degree n over a sub-field F of the field of all real numbers. Then there is an equation

$$f(x) = x^n + a_1x^{n-1} + \cdots + a_n = 0, \qquad (a_i \text{ in } F), \tag{1}$$

which is irreducible in F, and $K = F(X)$ consists of all polynomials with coefficients in F in an algebraic quantity X for which $f(x) = 0$. We call K a *total real* field over F if the ordinary complex roots

$$x_1, \cdots, x_n \tag{2}$$

of $f(x) = 0$ are all real. The modern theory of algebraic numbers has made the study of such fields of great interest.

A particular algebraic root of $f(x) = 0$ is given by the matrix

$$Y = \begin{pmatrix} 0 & 0 \cdots 0 & -a_n \\ 1 & 0 \cdots 0 & -a_{n-1} \\ 0 & 1 \cdots 0 & -a_{n-2} \\ \cdot & \cdot\cdot\cdot\cdot\cdot & \cdot \\ 0 & 0 \cdots 1 & -a_1 \end{pmatrix}. \tag{3}$$

This is a matrix whose characteristic equation is the above $f(x) = 0$. The irreducibility of $f(x)$ implies that every n-rowed square matrix Z with elements in F and $f(x) = 0$ as characteristic equation is similar to Y, and thus every such Z defines a field $F(Z)$ equivalent to K over F.

We shall obtain a normal form here for Z such that every Z in our form and with irreducible characteristic equation defines a total real field, while conversely every total real field is defined by one of our matrices. Our result will then provide *a construction of all total real fields over F*. The irreducibility condition is of course a part of the final conditions in all problems on the construction of algebraic fields and should not be considered as

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