any point of $U_m(b)$. Then $ax<2^{-m}$ and $xy<2^{-m}$ from the condition on the diameters of $U_m(a)$ and $U_m(b)$. Hence we have $ay<2^{1-m}<r$, and therefore y is in $S(a)$, which is contained in $U_n(a)$. Hence $U_m(b) \subset U_n(a)$.

The condition of this theorem has an advantage over the condition of Alexandroff and Urysohn. The latter condition postulates the existence of families of covering sets $\{G_n\}$ having certain properties, and in terms of these sets the distance function is defined. Given a neighborhood space, it might be difficult to determine whether such families of sets $\{G_n\}$ could be found. The condition of the present theorem also leads to the existence of such sets with the additional information that they are to be found among the original neighborhoods of the space.

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AN INVOLUTORIAL LINE TRANSFORMATION DETERMINED BY A CONGRUENCE OF TWISTED CUBIC CURVES*

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1. *Introduction.* Consider the pencils of quadric cones $K_1-\alpha K_2=0$, $L_1-\beta L_2=0$, each pencil having a common vertex which lies on all of the cones of the other pencil. For a given α, β the curve $C(\alpha, \beta)$ of intersection of the cones is composite, consisting of the line l of the vertices of the two pencils and a twisted cubic curve $C_3(\alpha, \beta)$. As α, β take on all values independently, $C_3(\alpha, \beta)$ describes a congruence of space cubic curves. An arbitrary line t of space will be bisecant to just one $C_3(\alpha, \beta)$, for any three points of space will determine a set of values for the parameters α, β, ρ in the system

$$(1) \qquad (K_1 - \alpha K_2) - \rho(L_1 - \beta L_2) = 0$$

of quadric surfaces, and if these three points be chosen on t, then t lies on the quadric of (1) so determined and will meet $C_3(\alpha, \beta)$ twice. We shall henceforth write $C_3(t)$ for this curve.

Now consider a fixed plane π and in this plane a Cremona involutorial transformation Γ of order n having a curve Δ_m of

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