ON CERTAIN ARITHMETIC FUNCTIONS OF SEVERAL ARGUMENTS*

BY LEONARD CARLITZ

1. *Introduction.* Series of the type

$$\sum_{l,m,n} \beta(l, m, n), \tag{1}$$

summed over all positive l, m, n satisfying the conditions

$$(m, n) = (n, l) = (l, m) = 1, \tag{2}$$

occur in a problem in additive arithmetic. The series (1) is transformed into a series $\sum \gamma(l, m, n)$, now summed over all positive l, m, n, where

$$\gamma(l, m, n) = \sum_{e,f,g=1}^{\infty} \mu(e, f, g)\beta(el, fm, gn).$$

The function $\mu(e, f, g)$ may be defined by

$$\sum \mu(e, f, g) = \begin{cases} 1 & \text{for } (m, n) = (n, l) = (l, m) = 1, \\ 0 & \text{otherwise}, \end{cases} \tag{3}$$

the summation on the left extending over all $e \mid l, f \mid m, g \mid n$.

In this note we define a class of functions μ satisfying relations of the type (3); the functions generalize, in several directions, the ordinary Möbius μ-functions. We next define and evaluate a class of generalized ϕ-functions; they may be expressed in terms of μ.

2. *The μ-Functions.* For arbitrary positive k, s we define the function $\mu^s(m_1, \cdots, m_k)$ by means of

$$\sum_{e_i \mid m_i} \mu^s(e_1, \cdots, e_k) = \begin{cases} 1 & \text{for } M^s, \\ 0 & \text{otherwise}, \end{cases} \tag{4}$$

the k-fold summation on the left extending over all $e_i \mid m_i$, $(i=1, \cdots, k)$, while M^s is an abbreviation for the $C_{k,s}$ simultaneous conditions

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