NOTE ON GROUP POSTULATES*

BY HERBERT BOGGS AND G. Y. RAINICH

1. *Introduction.* A note† by R. Garver in which he proves (following Huntington) that the existence of the product may be deduced from the existence of the right and left quotients and the associative law of multiplication suggests that it might be interesting to consider division (or the two divisions in the non-commutative case) as the fundamental operation, and to formulate a system of postulates in terms of that operation so that multiplication does not appear in the postulates at all. This naturally leads to a translation of the associative law of multiplication into a form which involves divisions only. That is what we do in the following paragraphs. A system of group postulates in terms of division has been given previously by Morgan Ward,‡ but Ward introduces only one division (let us say the right division) and as a result of that his system seems to lack symmetry and does not contain a direct equivalent of the associative law of multiplication.

2. *Proposed Postulates.* The postulates are as follows.

I. *To every two elements a and b of G there corresponds a unique element $c=a/b$, called the right quotient of a divided by b.*

II. *To every two elements a and b of G there corresponds a unique element $d=b\backslash a$, called the left quotient of a divided by b.*

III. *$a/b=c$ means the same as $c\backslash a=b$.*

IV. (*Associative law*) $(a\backslash b)/c=a\backslash(b/c)$.

The independence of this set (or rather of a set obtained from these postulates by slight modification to insure the possibility of considering each independently from the others) is proved very easily by exhibiting, as does Ward, realizations in which all but one of the postulates is satisfied. For instance, if we interpret G as the system of rational integers, a/b as the sum $a+b$, $c\backslash a$ as the difference $a-c$, Postulates I, II, IV are satisfied but Postulate III is not.

* Presented to the Society, April 10, 1936.

† This Bulletin, vol. 40 (1934), pp. 698–701.

‡ Transactions of this Society, vol. 32 (1930), pp. 520–526.