mediate functions and (2) the roots of $y_{n'}=0$. In the second set the contribution due to (1) persists. By the introduction of the additional functions in the second set, the change contributed by a root of $y_{n'}=0$ in the first set is transferred to a root of $y_{n''}=0$ in the second set. Also no change arises for a root of $y_{n'}=0$, $y_{n'+1}=0, \cdots, y_{n''-1}=0$. Since the total number of losses is the same in the two cases, the conclusion will be that the number of roots for $y_{n'}=0$ and $y_{n''}=0$ in the range $(-\infty, a)$ will be the same; similarly for the range (b, ∞). Also in (a, b) the numbers of roots of $y_{n'}=0$ and $y_{n''}=0$ are $n'-p-q$ and $n''-p-q$. Hence we conclude that the number of imaginary roots of $y_{n'}=0$ and $y_{n''}=0$ will be the same for $n''>n'\geqq n_0$.

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NOTE ON THE EXISTENCE OF AN nTH DERIVATIVE DEFINED BY MEANS OF A SINGLE LIMIT

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The nth derivative of a function $f(x)$ may be defined without the use of derivatives of lower order by means of the limit of a certain quotient. Conditions necessary and sufficient for the existence and continuity of $f^{(n)}(x)$ at a point $x=a$ and also for the mere existence of $f^{(n)}(a)$ have been recently given by Franklin.* The purpose of the present note is to state necessary and sufficient conditions of a somewhat more general form with proofs which use only Rolle's theorem and elementary properties of determinants.

Let $f_i(x)$ and $\phi_i(x)$, $(i=1, 2, \cdots, n+1)$, be functions possessing derivatives of the nth order, continuous in an interval I. Let $x_1, x_2, \cdots, x_{n+1}$ be points of I which close down in an arbitrary manner on a point a, in the sense that

$$(1) \qquad |x_j - a| < \epsilon_k, \qquad \lim_{k\to\infty} \epsilon_k = 0.$$

We shall use the notation

* This Bulletin, vol. 41 (1935), p. 573.