GENERALIZED JACOBI POLYNOMIALS

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1. *Introduction.* The differential equation

$$(\alpha x^2 + \beta x + \gamma)\frac{d^2y}{dx^2} - (x + a_1)\frac{dy}{dx} + \{n - n(n-1)\alpha\}y = 0,$$

where n is a positive integer, has polynomial solutions y_n of degree n. Some properties of these polynomials have been obtained by Brenke* and by Lawton.† The object of this paper is to derive fresh properties and in particular to study the zeros of these polynomials. Brenke proved that

$$h_n y_n = \frac{1}{\rho} D^n\{\rho P^n\},$$

where

$$P \equiv \alpha x^2 + \beta x + \gamma \equiv -\alpha(x-a)(b-x), \quad (a < b),$$

$$\rho \equiv -\frac{1}{\alpha}(x-a)^{A-1}(b-x)^{B-1},$$

$$A = \frac{a + a_1}{\alpha(b-a)}, \qquad B = \frac{b + a_1}{-\alpha(b-a)},$$

and h_n is the coefficient of x^n in the right-hand side. It has also been proved by him that if A and B are positive, the following recurrence formula holds good.

$$\text{(A)} \qquad y_n = (a_n + x)y_{n-1} - b_n y_{n-2},$$

where

$$b_n = \frac{c_{n-2}^2}{c_{n-1}^2} \quad \text{and} \quad \frac{1}{c_n^2} = \int_a^b \rho y_n^2\, dx,$$

$$a_n = -c_{n-1}^2 k_{n-1}, \quad \text{where} \quad k_n = \int_a^b x\rho y_n^2\, dx.$$

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* This Bulletin, vol. 36 (1930).

† This Bulletin, vol. 38 (1932).