A MIXTURE THEOREM FOR NONCONSERVATIVE MECHANICAL SYSTEMS*

BY A. H. COPELAND†

We shall be concerned with a mechanical system which is acted upon by friction in such a manner that it eventually comes to rest. We shall assume that there is a probability distribution for the initial conditions of the system. There is then determined a corresponding distribution for the rest position. We shall show that in general as the friction approaches zero, this latter distribution approaches uniformity irrespective of the initial distribution. This type of problem was first solved by Poincaré.‡ The theory was later developed by Smoluchowski§ and recently an extensive contribution based on the ergodic theorem|| has been made by Hopf.¶

We shall let $x_1, x_2, \cdots, x_\nu$ denote the position coordinates of the system at any time t and shall let x denote the vector $x_1, x_2, \cdots, x_\nu$. Thus dx/dt is the velocity vector. The initial position will be denoted by the vector B and the initial velocity by the vector V. Then V, B can be any point of 2ν-dimensional space Ω. We assume that there exists a continuous function ψ such that the probability of the initial conditions being represented by a point V, B in a given region S is equal to the integral of ψ taken over S. This condition is expressed by the equation

$$\text{prob }[(V, B)\epsilon S] = \int_S \psi(V, B)d\omega. \tag{1}$$

* Presented to the Society, December 27, 1934.

† Guggenheim fellow.

‡ *Calcul des Probabilités*, 1912.

§ *Über den Begriff des Zufalls und den Ursprung der Wahrscheinlichkeitsgesetze in der Physik*, Die Naturwissenschaften, vol. 6 (1918).

|| Birkhoff, *Proof of a recurrence theorem for strongly transitive systems*, *Proof of the ergodic theorem*, Proceedings of the National Academy of Sciences, vol. 17 (1931).

¶ *On causality, statistics, and probability*, Journal of Mathematics and Physics, Massachusetts Institute of Technology, vol. 13 (1934). See also Struik, *On the foundations of the theory of probability*, Philosophy of Science, vol. 1 (1934).