REMARKS ON THE INDUCTIVE PRINCIPLE AND RELATED EXISTENCE THEOREMS†

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It is the purpose of the present note to show the validity, in a natural formulation, of the inductive principle and of related existence theorems for the general linear order and for the n-fold order.‡ The simple relations which these theorems bear to the inductive principle and to one another are brought out tersely. An example shows that the inductive principle is not valid for the general $\aleph_0$-fold order.

Let A be a given linear order. By an *extension of an initial segment* I of A—analog of neighborhood of a point—we understand a segment§ of A containing elements of both|| I and $\bar{I}$. If E ($\neq A$) is a subset of A, let I be the set of elements of A preceding all the elements of E; we refer to I as *the initial segment associated with E*. While I consists exclusively of elements of $\bar{E}$, every extension of it contains elements of E. To deny the existence of an initial segment I thus characterized is to affirm that $\bar{E}\equiv A$. In other words, to say that for every initial segment consisting exclusively of elements of $\bar{E}$ there is an extension consisting exclusively of elements of $\bar{E}$ is to say that $\bar{E}\equiv A$. The inductive principle for a general linear order is thus a rewording of the characteristic property of the initial segment

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‡ The inductive principle appears here in a different light from that of the view represented, for example, by H. Poincaré in *Science and Hypothesis*. In this connection, see also Khintchine, *Das Stetigkeitsaxiom des Linearcontinuums als Inductionsprinzip betrachtet*, Fundamenta Mathematicae, vol. 4 (1923), p. 164, and Hildebrandt, *The Borel theorem and its generalizations*, this Bulletin, vol. 32 (1926), p. 423.

§ A segment of A is a subset of A containing with every pair of its elements all the elements of A between them.

|| We denote the complement $A-E$ of a set E by $\bar{E}$. If $I=0$, where 0 stands for the null initial segment, we regard every non-null initial segment of A as an extension of I; if $I=A$, we regard every non-null final segment of A as an extension of I.