ALEXANDROFF AND HOPF ON TOPOLOGY

Topologie I. By Paul Alexandroff and Heinz Hopf. Berlin, Springer, 1935. xiii+636 pp.

Topology is not a young subject; this year may be described as the two hundredth anniversary of its birth if we agree that it had its beginning in the problem of the seven bridges of Königsberg settled by Euler in 1736. But the systematic development of topology is new; it has only come since the work of Poincaré at the turn of the century. The International Topological Conference held at Moscow last September showed that the subject has attained a definite measure of maturity and a wide range of influence on other branches of mathematics, but that it is still undergoing rapid growth and flux. Just when topological activity seems to be slackening some new point of view sets it seething again; in the past year we have had an example of this in the "dual cycles" of Alexander and Kolmogoroff.

Books on topology are so few that the appearance of a new one is an important event. The volume now before us is especially impressive, not only for its own size and thoroughness, but because it is the first of three volumes which together are intended to give a detailed survey of topology as a whole. The authors are distinguished geometers who have been in close contact with the various topological centers in Europe and America. They began their difficult task several years ago at the suggestion of Courant, under whose editorship the book appears as the forty-fifth of the familiar Springer series. Many of us have known about the project and have expectantly awaited the finished work.

In their attempt to treat topology as a whole the authors have steered a middle course between point-set theory on the one hand and algebraic topology on the other. They do this quite easily since in their own investigations they both have used combinatorial methods to solve set-theoretic problems; no better example of such blending could be cited than Alexandroff's theory of dimension. As the central concept for the first volume they have chosen the (euclidean) *polyhedron*, a subset of multi-dimensional euclidean space which can be partitioned into flat convex cells in an orthodox manner. The polyhedron holds in their program an intermediate position between the generality of an *abstract* space and the special character of a *manifold;* by appropriate approximation, for example, one passes from polyhedra to compact metric spaces, and by suitable specialization one passes from a polyhedron to a manifold. In accordance with this plan the second volume is to deal with set-theoretic questions, such as dimensionality, and the third with manifolds.

The book (that is, the first volume) falls into four parts. Part I is a hundred-page introduction to abstract-space topology; much of this has no bearing on the subsequent study of polyhedra but it does have an essential place in the general program of the three volumes. The first chapter begins with various methods of topologization, closure, limits, distance, neighborhoods, and so on, then passes to separation and countability axioms, and finally to the Urysohn theorem that a normal space with countable basis can be immersed in Hilbert