ON THE MODULUS OF THE DERIVATIVE OF A POLYNOMIAL*

BY W. E. SEWELL

1. *Introduction.* Let $P_n(z)$ be an arbitrary polynomial of degree n in z and let $|P_n(z)| \leqq M$ on a set C. The modulus of $P_n'(z)$† on C has an upper bound depending on M, on n, and on the set C. In this connection A. Markoff‡ has proved the following theorem.

Let $|P_n(z)| \leqq 1$ *in the interval* $-1 \leqq z \leqq +1$. *Then* $|P_n'(z)| \leqq n^2$ *for* $-1 \leqq z \leqq +1$. *This bound is attained only by the polynomial* $\pm \alpha \cos n \operatorname{arc} \cos z$, $|\alpha| = 1$.

A second fundamental result is the following theorem of S. Bernstein.§

Let $|P_n(z)| \leqq 1$ *on* C: $|z| \leqq 1$. *Then* $|P_n'(z)| \leqq n$ *on* C. *This bound is attained only by the polynomial* αz^n, $|\alpha| = 1$.

These theorems have been generalized in various directions by P. Montel,|| G. Szegö,¶ Dunham Jackson,** and the author.†† Here we will prove the following generalization.

THEOREM A. *Let* $P_n(z)$, *a polynomial of degree* n *in* z, *be in modulus less than a constant* M *on a set* C *which has no isolated points and whose complement has finite connectivity. Then*

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† $P_n'(z)$ denotes the first derivative of $P_n(z)$.

‡ A. Markoff, Abhandlungen der Akademie der Wissenschaften zu St. Petersburg, vol. 62 (1889), pp. 1–24. Markoff considers only polynomials with real coefficients. For the general case see M. Riesz, Jahresbericht der Deutschen Mathematiker-Vereinigung, vol. 23 (1914), pp. 354–368; see especially p. 357.

§ S. Bernstein, *Leçons sur les Propriétés Extrémales*, 1926, pp. 44–46.

|| P. Montel, Bulletin de la Société Mathématique de France, vol. 46 (1919), pp. 151–196.

¶ G. Szegö, Mathematische Zeitschrift, vol. 23 (1925), pp. 45–61.

** Dunham Jackson, this Bulletin, vol. 36 (1930), pp. 851–857; vol. 37 (1931), pp. 883–890.

†† W. E. Sewell, Proceedings National Academy of Sciences, vol. 21 (1935), pp. 255–258.