If m_1 and hence m_{k-1} is positive, (9) tells us that all the m_{k-1} members of S: $e_{s_{k-1}+1}, \cdots, e_{s_{k-1}+m_{k-1}}$ belong to C. In particular the last one which is e_{s_k} belongs to C. Moreover $m_k = n - \theta(s_k) = 0$. Hence e_{s_k} is the nth member of C. If m_1 and hence m_{k-1} is negative, (9) tells us that all the m_{k-1} members of S: $e_{s_k+1}, \cdots, e_{s_{k-1}}$ belong to C. In particular $e_{s_{k-1}}$ is a member of C. But $m_{k-1} = n - \theta(s_{k-1})$ or $\theta(s_{k-1}) = n - m_{k-1}$; hence $e_{s_{k-1}}$ is the $(n - m_{k-1})$st member of C. Thus the proof is complete.

If S and C are wholly arbitrary one may show by means of examples that the conclusion of the theorem is the best that can be obtained from the hypothesis. Additions to the hypothesis make the last two statements of the theorem more precise. For example, if we assume that no two consecutive members of C are consecutive members of S, it is easy to prove that $e_{s_{k-1}}$ is the $(n-1)$st member of C in case $m_1 < 0$.

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A NEW CLASS OF FUNCTIONS OF TWO VARIABLES INVOLVING BESSEL FUNCTIONS OF HALF AN ODD INTEGER*

BY N. A. HALL

The evaluation of certain integrals arising in the theory of the conduction of heat between two media of different conductivities† suggested the consideration of the expansions:

$$\sin\left[\alpha(1 + x^2)^{1/2} - \beta x\right] = \sum_{n=0}^{\infty} x^n S_n(\alpha, \beta), \tag{1}$$

$$\cos\left[\alpha(1 + x^2)^{1/2} - \beta x\right] = \sum_{n=0}^{\infty} x^n C_n(\alpha, \beta), \tag{2}$$

$$\exp i\left[\alpha(1 + x^2)^{1/2} - \beta x\right] = \sum_{n=0}^{\infty} x^n E_n^{(1)}(\alpha, \beta), \tag{3}$$

$$\exp - i\left[\alpha(1 + x^2)^{1/2} - \beta x\right] = \sum_{n=0}^{\infty} x^n E_n^{(2)}(\alpha, \beta). \tag{4}$$

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* Presented to the Society, November 30, 1935.

† G. Green, Philosophical Magazine, (7), vol. 18 (1934), p. 631.