A GENERALIZED INVERSIVE ALGORITHM

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In a previous note* we considered an algorithm for constructing the nth member of any infinite class C of positive integers, an algorithm used by Brun† for the non-tentative calculation of the nth prime. In this note we give a generalization in two directions of the Brun algorithm. In the first place it is made applicable to any denumerable set of elements and secondly a parameter (m_0) is introduced which allows the algorithm to start from a more advantageous position than in the case of Brun, where one starts by saying that the nth prime is not less than n.

The algorithm may be presented in the following form:

Let $S: e_1, e_2, e_3, \cdots$, be a class of ordered elements.

Let $C: e_{a_1}, e_{a_2}, e_{a_3}, \cdots, (a_i < a_{i-1})$, be any infinite subclass of S. For x an integer‡ >0, we define $\theta(x)$ as the number of elements of C belonging to the set of the first x elements of S. For $x \leqq 0$, we define $\theta(x) = 0$. Let m_0 and n be any positive integers. We then form the sequence

$$m_1, m_2, m_3, \cdots, \tag{1}$$

defined by

$$\begin{aligned} m_1 &= n - \theta(m_0), \\ m_2 &= n - \theta(m_0 + m_1), \\ m_3 &= n - \theta(m_0 + m_1 + m_2), \\ &\cdots\cdots\cdots\cdots \\ m_r &= n - \theta(s_r), \end{aligned} \tag{2}$$

where we have written s_r for $m_0 + m_1 + \cdots + m_{r-1}$. We may then state the following theorem.

THEOREM. *The terms of* (1) *do not differ in sign, nor do they increase in absolute value. There exists moreover an integer k such*

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* This Bulletin, vol. 38 (1932), pp. 693–694.

† Kongelige Norske Videnskaps-selskabet, vol. 4, pp. 66–69.

‡ If x is not an integer one could define $\theta(x) = \theta([x])$.