ON THE PRINCIPLES OF HAMILTON AND CARTAN
(SUPPLEMENTARY NOTE)

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A request from Dr. D. C. Lewis for an explanation regarding a transformation which appeared in my paper on this topic (this Bulletin, February, 1936) led me on examination to find that the results there given are true in a restricted case but not in general. They are valid when the a_{rs} and the a_s of the non-holonomic relations are functions of t only. Also, under these conditions it can be shown that the order of integration in (15) and (16) is reversible and that then the action integral

$$\int_{u_0}^{u_1} \sum_1^k p_r dq_r - H dt$$

is an extremal for an arc of a trajectory as compared with all neighboring paths which are kinematically possible.

There is, however, a coordination of the principles for the general case. For, using the notation which was adopted, consider the expression

$$\sum_1^k \left\{ \left(dq_r - \frac{\partial H}{\partial p_r} dt \right) \delta p_r + \left(- dp_r - \frac{\partial H}{\partial q_r} dt + \sum_1^m a_{rs}\lambda_s dt \right) \delta q_r + \left(dH - \frac{\partial H}{\partial t} dt + \sum_1^m a_s \lambda_s dt \right) \delta t \right\}. \tag{17}$$

The integral of (17) over a Cartan locus can, after an integration by parts, be written as the negative of

$$d \int_{\alpha_0}^{\alpha_1} \sum_1^k p_r \delta q_r - H\delta t - \int_{\alpha_0}^{\alpha_1} \sum_1^m \lambda_s \left(\sum_1^k a_{rs}\delta q_r + a_s \delta t \right) dt, \tag{18}$$

and its integral over an arc of a trajectory is

$$\delta \int_{u_0}^{u_1} \sum_1^k p_r dq_r - H dt + \int_{u_0}^{u_1} \sum_1^m \lambda_s \left(\sum_1^k a_{rs}\delta q_r + a_s \delta t \right) dt. \tag{19}$$