Each normal  $C_n$  image of a line meets each base  $S_{n-2}$  in n-2 points and does not intersect the ruled variety.

The images of planes intersect R in (n+1)(n-2)/2 lines. The plane meets each base  $S_{n-2}$  in a point, the image of which is a line meeting n of the base  $S_{n-2}'$  and lying on  $F_2$ . Each base  $S_{n-2}$  meets R in a manifold of dimensionality n-3 and of order n-1. For n=4, the two-dimensional variety of order 5 has an infinite number of plane elliptic cubic curves, but the corresponding property is not true for larger values of n although the intersections of each base  $S_{n-2}$  and R are birationally equivalent.

CORNELL UNIVERSITY AND WELLS COLLEGE

## ON THE CHARACTERISTIC ROOTS OF MATRIC POLYNOMIALS\*

## BY N. H. McCOY

1. Introduction. Unless otherwise stated, all matrices and polynomials are assumed to have coefficients in an arbitrary algebraically closed field K.

Let A and B denote square matrices of order n. If the characteristic roots of every polynomial f(A, B) are all of the form  $f(\lambda, \mu)$ , where  $\lambda$  and  $\mu$  are characteristic roots of A and B, respectively, then in accordance with a notation to be introduced below, we shall say that the matrices A, B have property  $I_n$ . By a theorem of Frobenius,  $\dagger$  the matrices A, B have this property if they are commutative, but this is by no means a necessary condition. The study of pairs of matrices having property  $I_n$  has been the subject of papers by Bruton, Ingraham, and Roth.  $\ddagger$  However, in no case have conditions been obtained which are both necessary and sufficient for the existence of this property.

<sup>\*</sup> Presented to the Society, October 26, 1935.

<sup>†</sup> G. Frobenius, Über verlauschbare Matrizen, Sitzungsberichte der Preussischen Akademie der Wissenschaften zu Berlin, 1896, pp. 601-614.

<sup>‡</sup> The papers by Bruton and by Ingraham have not yet been published in full but abstracts are available as follows: G. S. Bruton, Certain aspects of the theory of equations for a pair of matrices, this Bulletin, vol. 38 (1932), p. 633; M. H. Ingraham, A study of certain related pairs of square matrices, this Bulletin, vol. 38 (1932), pp. 633-634. Roth's paper is On the characteristic values of the matrix f(A, B), Transactions of this Society, vol. 39 (1936), pp. 234-243.