A GENERALIZATION OF A CYCLOTOMIC FORMULA*

BY H. S. GRANT

1. *Introduction*. Jacobi stated without proof† the following cyclotomic formula

$$F(-1)\cdot F(\alpha^2) = \alpha^{2m}F(\alpha)\cdot F(-\alpha),$$

where

$$F(\alpha) = x + \alpha x^g + \alpha^2 x^{g^2} + \cdots + \alpha^{q-2}x^{g^{q-2}},$$

q is an odd prime, g a primitive root, mod q, $g^m \equiv 2$, mod q, $x^q = 1(x \neq 1)$, $\alpha^{q-1} = 1(\alpha \neq 1)$. This relation is essentially one involving Lagrange resolvent functions, and ultimately reduces to one connecting two Jacobi ψ-functions. The former have been generalized by L. Stickelberger,‡ and the latter by H. H. Mitchell.§

It is the purpose of this paper to generalize Jacobi's formula to the case $q^t \equiv 1$, mod n, q an odd prime, t any exponent for which the congruence holds, n even. If we take $t=1$, $n=q-1$, the relation stated above then follows as a special case. Before proceeding further, the reader is strongly advised to refer to Mitchell's paper mentioned above, frequent use of which is made in what follows.

2. *Characteristic Properties of the Generalized Function*. If $s(x) = \sum_{i=0}^{t-1} a_i x^i$, a_i reduced, mod q, q prime, $s(q)$ will represent a complete residue system, mod q^t. We interpret $s(x)$ as the marks of a Galois field of order q^t. Let ϵ denote a primitive nth root of unity, where $q^t \equiv 1$, mod n, and τ a primitive q^tth root of unity. We define

$$(\epsilon^\lambda, \tau) = \sum_s \epsilon^{\lambda \text{ ind } s(x)} \tau^{s(q)},$$

the summation being taken over all marks excepting 0, and the

* Presented to the Society, September 13, 1935.

† Journal für Mathematik, vol. 30 (1846), p. 167.

‡ Mathematische Annalen, vol. 37 (1890), pp. 321–367.

§ Transactions of this Society, vol. 17 (1916), pp. 165–177.