PRODUCTS OF METHODS OF SUMMABILITY*

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1. *Introduction.* Let the transforms

$$A: \qquad \sigma_n = \sum_{k=1}^{\infty} a_{nk}s_k,$$

$$B: \qquad \tau_n = \sum_{k=1}^{\infty} b_{nk}s_k,$$

be two regular† methods of summability. Then the A transform $\{\sigma_n\}$ of the B transform $\{\tau_n\}$ of a sequence $\{s_n\}$ is (if it exists) given by

$$(1) \qquad \sigma_n = \sum_{p=1}^{\infty} a_{np}\tau_p = \sum_{p=1}^{\infty}\sum_{k=1}^{\infty} a_{np}b_{pk}s_k.$$

If $\{s_n\}$, bounded or not, is summable B to L so that $\tau_n \rightarrow L$, then regularity of A implies that $\{\sigma_n\}$ exists and $\sigma_n \rightarrow L$ as $n \rightarrow \infty$.

If the sequence $\{s_n\}$ is *bounded*, then the last series in (1) converges absolutely (as a double series) and we can reverse the order of summation to obtain

$$(2) \qquad \sigma_n = \sum_{k=1}^{\infty}\left\{\sum_{p=1}^{\infty} a_{np}b_{pk}\right\} s_k.$$

The matrix $||c_{nk}|| \equiv ||\sum a_{np}b_{pk}||$ of (2) is the ordinary matrix product $||a_{nk}||\ ||b_{nk}||$ and the transformation

$$AB: \qquad \omega_n = \sum_{k=1}^{\infty} c_{nk}s_k$$

is denoted by AB as indicated. We shall show that for regular infinite matrices A, B, it may not be true that $AB \supset B$; and that regularity of A, B, D and equivalence of A and D do not necessarily imply equivalence of AB and DB. We give also related results and applications to kernel transformations.

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† The terminology and facts relating to summability which we use are to be found in the expository paper, *Report on topics in the theory of divergent series*, by W. A. Hurwitz, this Bulletin, vol. 28 (1922), pp. 17–36.