Two rational automorphs A_1 and A_2 of f may be called *right-equivalent* if there exists an integral automorph I of f such that $A_1I=A_2$. If (T_1, U_1) and (T_2, U_2) belong to the same set* of solutions of (5), the corresponding automorphs (with $t=T_i/m$, $u=U_i/m$) are readily seen to be right-equivalent.

McGill University

A NOTE ON RECURSIVE FUNCTIONS†

BY S. C. KLEENE

The notion of a recursive function of natural numbers, which is familiar in the special cases associated with primitive recursions, Ackermann-Péter multiple recursions, and others, has received a general formulation from Herbrand and Gödel. The resulting notion is of especial interest, since the intuitive notion of a "constructive" or "effectively calculable" function of natural numbers can be identified with it very satisfactorily.‡ Consider the operation of passing from a function $\rho(x_1, \cdots, x_n, y)$, such that for each set of values of $x_1, \cdots, x_n$ the equation $\rho(x_1, \cdots, x_n, y)=0$ has solutions for y, to the function "$\epsilon y[\rho(x_1, \cdots, x_n, y)=0]$" of which the least solution is $x_1, \cdots, x_n$. We have shown that the (general) recursive functions are the functions which are derivable from the primitive recursive functions by one application of this operation and of substitution.§ Herein we note the related result, that the recursive functions are the functions obtainable by repeated applications of the operation just described and of substitution from the three particular functions $x+y$ (sum), $x\cdot y$ (product), δ_y^x (Kronecker delta). This result follows from the other by an adaptation of an argument used by Gödel in proof that every

---

* For definition of set, see Pall, Transactions of this Society, vol. 35 (1933), p. 491; or Dirichlet, *Vorlesungen über Zahlentheorie*, §87.

† Presented to the Society, January 1, 1936.

‡ See A. Church, *An unsolvable problem of elementary number theory*, American Journal of Mathematics, vol. 58 (1936), pp. 345–363, §7.

§ S. C. Kleene, *General recursive functions of natural numbers*, Mathematische Annalen, vol. 112 (1936), No. 5, IV and V.