A GENERALIZATION OF WARING'S PROBLEM*

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1. *Introduction.* Define $g(n, m)$ so that every integer $\geqq m$ is a sum of g nth powers, while not every integer $\geqq m$ is a sum of $g-1$ powers. It is customary to write $g(n)$ for $g(n, 0)=g(n, 1)$. Quite recently I evaluated $g(n)$ for every $n>6$.

For $n=9$ or 11, I evaluate $g(n, m)$ for each m below specified large values M. In particular, $g(11, M)=336$ and $g(9, M)=163$ are small compared with $g(11)=2132$ and $g(9)=548$.

By use of the Hardy-Littlewood Theory and extensive tables, it was found that $g(6)\leqq 160$. I here obtain $g(6)\leqq 110$.

2. *Asymptotic Theory.* Recently I proved† that, if $n\geqq 4$, every integer $\geqq N$ is a sum of $s-2+3k$ integral nth powers $\geqq 0$, where the quantities are defined as follows. Let p^θ be the highest power of the prime p which divides n. Write $\gamma=\theta+1$ if $p>2$, $\gamma=\theta+2$ if $p=2$. Let D be the g.c.d. of $p-1$ and n/p^θ. Write $m=D(p^\gamma-1)/(p-1)$. Then the conditions on s (for Lemma B of A) are $s>2n$, $s\geqq m+1$, for every prime p. They hold for $s\geqq 13$ if $n=6$, and for $s\geqq 2n+1$ if n is an odd prime or its square.

Employ natural logarithms. If $\vartheta(b)$ is the sum of the logarithms of all primes $\leqq b$,

$$\vartheta(b) \leqq \frac{6}{5}(.92129)b + 3\log^2 b + 8\log b + 5.$$

Take

$$\begin{aligned} b &= (1+n^s)^{2/(s-5)},\\ -\log c &= \log n + (n+1)\log 3 + 2n^2\log 2 + n(2n-1)\vartheta(b),\\ C &= 12(8n\cdot 3^{n-1})^{1/2}2^{(n-1)/4}n^{3(n-1)/2}/(3/2)^{(1-1/n)/2},\\ N^{-J/n} &= C/c, \quad 2J = \sigma\left(3-\frac{1}{2n}\right) + z - \frac{(n-1)}{2n^2}, \end{aligned}$$

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* Presented to the Society, October 26, 1935.

† Annals of Mathematics, vol. 37 (1936), pp. 293–316, cited as A; American Journal of Mathematics, July, 1936, cited as J.