A CHARACTERIZATION OF MANIFOLD BOUNDARIES IN E_n DEPENDENT ONLY ON LOWER DIMENSIONAL CONNECTIVITIES OF THE COMPLEMENT*

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In my recent paper *Generalized closed manifolds in n-space*† it was shown‡ that a compact point set B in E_n, common boundary of (at least) two domains D and D_1 which are respectively u.l.i-c.§ for $0 \leqq i \leqq j$ and $0 \leqq i \leqq n-j-3$ (where $n-2 > j \geqq (n-3)/2$), and such that the Betti numbers $p^{j+1}(D)$, $p^{j+2}(D), \cdots, p^{n-2}(D)$ are finite, is a g.c.$(n-1)$-m. This constituted a generalization of a former result|| to the effect that when $n=3$, D and D_1 are u.l.0-c., and $p^1(D)$ is finite, B is a closed 2-manifold. In the present note I propose to show, as principal result, that the above conditions on the numbers $p^{j+2}(D), \cdots, p^{n-2}(D)$ are irrelevant, and furthermore that it is immaterial whether we place the restriction as to finiteness on $p^{j+1}(D)$ or on $p^{n-j-2}(D_1)$. It turns out that the only essential requirements are that the upper limits on the dimensions for which D and D_1 are u.l.i-c. must total at least $n-3$, and that one of the domains have a finite Betti number as just stated.

For the sake of brevity we make the following definitions. We shall understand without explicit statement hereafter that the imbedding space is $E_n (n \geqq 3)$ (euclidean space of n dimensions).

DEFINITION. A metric space will be said to be *completely i-avoidable*¶ at a point P if for every $\epsilon > 0$ there exist δ and η, $\epsilon > \delta > \eta > 0$, such that if γ^i is a cycle on $F(P, \delta)$, then $\gamma^i \sim 0$ on $S(P, \epsilon) - S(P, \eta)$.

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* Presented to the Society, December 29, 1934.

† Annals of Mathematics, vol. 35 (1934), pp. 876–903; to be referred to hereafter as G.C.M.

‡ Principal Theorem E of G.C.M.

§ u.l.i-c. = uniformly locally i-connected; see G.C.M. for definition.

|| R. L. Wilder, *On the properties of domains and their boundaries in E_n*, Mathematische Annalen, vol. 109 (1933), pp. 273–306, Theorem 20; to be referred to hereafter as D.B.

¶ See condition (3), definition M^n, of G.C.M.