A REMARK ON THE ODD SCHLICHT FUNCTIONS

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Let (S) denote the class of analytic functions

$$f(z) = z + a_2z^2 + \cdots + a_nz^n + \cdots \tag{1}$$

regular and univalent or schlicht for $|z| < 1$, and (U) the subclass of odd schlicht functions

$$\phi(z) = [f(z^2)]^{1/2} = z + b_3z^3 + b_5z^5 + \cdots . \tag{2}$$

If $\phi(z)$ is real on the real axis, it has been shown* by J. Dieudonné that for all n

$$|b_{2n-1}| + |b_{2n+1}| \leqq 2, \qquad |b_3| \leqq 1. \tag{3}$$

This is not known to be true in the case where the coefficients are complex except for $n=1$. For complex coefficients it is known† that

$$|b_3| \leqq 1, \qquad |b_5| \leqq e^{-2/3} + \frac{1}{2} \qquad (> 1), \tag{4}$$

from which we could conclude only that

$$|b_3| + |b_5| \leqq \frac{3}{2} + e^{-2/3} \qquad (> 2).$$

It is the purpose of this paper to establish the inequality (3) for $n=2$ for the case when the coefficients are complex numbers; and to show further that

$$\frac{|b_3| + |b_5|}{2} \leqq \left(\frac{|b_3|^2 + |b_5|^2}{2}\right)^{1/2} \leqq 1, \tag{5}$$

$$|a_3| \leqq 1 + |b_3|^2 + |b_5|^2 \leqq 3. \tag{6}$$

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* See J. Dieudonné, Annales de l'Ecole Normale Supérieure, vol. 48 (1931), p. 318.

† See M. Fekete and G. Szegö, Journal of the London Mathematical Society, vol. 8 (1933), pp. 85–89.