CONVEX EXTENSION AND LINEAR INEQUALITIES*

BY L. L. DINES

A few years ago, at the Des Moines meeting, it was my privilege to address the Society on the subject *Linear inequalities*. In its simplest form the problem there considered had to do with a system of conditions

$$(1) \qquad \sum_{j=1}^{n} a_{ij}x_j > 0, \qquad (i = 1, 2, \cdots, m),$$

the coefficients a_{ij} being given real constants, and the x_j unknowns to be determined so as to satisfy the given conditions. The treatment was entirely analytic, and the aim was to develop a theory dictated by analogy with the theory of linear equations.

Today my purpose is to focus attention on geometric aspects of the theory, and in particular to show its close relationship to a certain geometric notion which in recent years has been useful in many investigations in analysis.

Let us consider an n-dimensional euclidean space $\mathfrak{U}$ of points $u \equiv (u_1, u_2, \cdots, u_n)$. A set of points in $\mathfrak{U}$ is said to be *convex* if the membership of two points $u^{(1)}$ and $u^{(2)}$ in the set implies the membership of all points on the line segment joining $u^{(1)}$ and $u^{(2)}$. A given set may or may not be convex, but any set $\mathfrak{M}$ may be *extended* so as to be convex by adjunction of the necessary points. The resulting set, which may be defined logically as the greatest common subset of all the convex sets which contain $\mathfrak{M}$, will be called the *convex extension* of $\mathfrak{M}$ and denoted by $C(\mathfrak{M})$.

This extended set, under various names and definitions, has been the subject of considerable study, and has been useful in many applications. One may refer to Minkowski's *Geometrie der Zahlen*, 1910; to Carathéodory's paper *Ueber der Variabilitätsbereich der Fourierschen Konstanten* in the Rendiconti del Circolo Matematico di Palermo (vol. 32 (1911)); or to the recent com-

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