

surability is infinitely improbable and that therefore  $N_i(t)$  is not periodic in practice implies the existence of a criterion of probability for the  $n/2$  periods, although such a criterion would necessarily be less elementary than the criterion of periodicity of  $N_i(t)$ —which is presumably estimated directly from the biological system.\*

This criticism is on something irrelevant to the real purpose of the book, as stated by the author in the sentence quoted above. The mathematical content of the book, a detailed analysis of certain differential and integro-differential equations, should be valuable both to the mathematician and to the statistician interested in the qualitative and quantitative study of the development of a biological system.

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*Interpolatory Function Theory.* By J. M. Whittaker. (Cambridge Tracts in Mathematics and Mathematical Physics, No. 33.) Cambridge University Press. New York, Macmillan, 1935. vii+107 pp.

This interesting tract by the son of E. T. Whittaker is devoted to the following problem. Let  $\Pi_i f(0)$  denote the differential operator

$$\Pi_i f(0) = \sum_{j=0}^{\infty} \frac{\pi_{ij}}{j!} f^{(j)}(0).$$

What analytic functions  $f(z)$  are uniquely determined by the values of  $\Pi_i f(0)$ ,  $i=0, 1, 2, 3, \dots$ ? This problem contains a large number of important special cases such as expansion in Taylor's series, interpolation at positive integers or at lattice points, and Bernoullian series. The fundamental concept is the notion of basic sets. A set of polynomials

$$p_i(z) = \sum_{j=0}^{\infty} p_{ij} z^j$$

is said to be *basic* if every polynomial is a linear combination of a finite number of polynomials from the set. A necessary and sufficient condition that the set be basic is that the matrix  $P = \|p_{ij}\|$  have a row-finite inverse. A set of operators is basic if the set of associated functions

$$p_i(z) = \sum_{j=0}^{\infty} \pi_{ji} z^j$$

is a basic set of polynomials, that is, if the transpose of the matrix  $\Pi = \|\pi_{ij}\|$  is row-finite and has a row-finite inverse. The series

$$\sum_{i=0}^{\infty} \Pi_i f(0) p_i(z)$$

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\* There seems to be a general principle in many investigations that the constants which occur in it have a continuous probability distribution, so that it is "infinitely improbable" that these constants have any preassigned set of values. As in the case just discussed, this unjustified assumption may be almost precisely what was to be proved.