# ON CERTAIN TWO-POINT EXPANSIONS OF INTEGRAL FUNCTIONS OF EXPONENTIAL TYPE

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1. *Introduction.* Hillel Poritsky* and G. J. Lidstone† found the following formal expansion

$$(1) \qquad f(x) = \sum_{n=0}^{\infty} f^{(2n)}(1)\Lambda_n(x) - \sum_{n=0}^{\infty} f^{(2n)}(0)\Lambda_n(x-1),$$

where $\Lambda_n(x)$ are polynomials (of degree $2n+1$) defined by the generating function

$$\sinh xt \operatorname{cosech} t = \sum_{n=0}^{\infty} t^{2n}\Lambda_n(x).$$

Expansion (1) holds for any polynomial $f(x)$ and solves formally the interpolation problem

$$(2) \qquad f^{(2n)}(1) = a_n, \qquad f^{(2n)}(0) = b_n, \qquad (n \geqq 0).$$

An integral function $f(x)$ is said to be of exponential type if the quantity

$$(3) \qquad \gamma(f) = \overline{\lim_{r\to\infty}} \frac{\log M(r)}{r},$$

which is called the type of the function $f(x)$, is finite, $M(r)$ denoting the maximum modulus of $f(x)$ on the circle $|x| = r$.

Poritsky and J. M. Whittaker‡ proved that the expansion (1)

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* Hillel Poritsky, *On certain polynomial and other approximations to analytic functions*, Transactions of this Society, vol. 34 (1932), pp. 274–331.

† G. J. Lidstone, *Notes on the extension of Aitken's theorem (for polynomial interpolation) to the Everett types*, Proceedings Edinburgh Mathematical Society, vol. 2 (1930), pp. 16–19.

‡ J. M. Whittaker, *On Lidstone's series and two-point expansions of analytic functions*, Proceedings London Mathematical Society, vol. 36 (1933–34), pp. 451–469. In order to facilitate reference we use throughout Whittaker's notations.