ON TRANSFORMATIONS OF DOUBLE SERIES

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1. *Introduction and Definition of Notation.* A double series $\sum_{k,l=1}^{\infty} x_{kl}$ may be classified according to the behavior of the double sequence of its partial sums $s_{kl} \equiv \sum_{i,j=1}^{k,l} x_{ij}$ as follows. The sequence $\{s_{kl}\}$ is *ultimately bounded* (abbreviated *ub*) if there exists a number Q such that s_{kl} is bounded for all $k, l > Q$; *bounded* (*b*) if in the preceding case Q can be taken to be zero; *convergent* (*c*) if $\lim_{k,l\to\infty} s_{kl}$ exists (finite); *bounded convergent* (*bc*) if both *b* and *c*; *ultimately regularly convergent* (*urc*) if *c*, and if there exists a number $\bar{Q}$ such that $\lim_{k\to\infty} s_{kl}$ and $\lim_{l\to\infty} s_{kl}$ both exist (finite) for all $l > \bar{Q}$ and all $k > \bar{Q}$, respectively; *regularly convergent* (*rc*) if in the preceding case $\bar{Q}$ can be taken to be zero; *bounded ultimately regularly convergent* (*burc*) if both *b* and *urc*.

It is the purpose of the present paper to establish necessary and sufficient conditions on the matrix $\|b_{kl}\|$ in order that, whenever the series $\sum_{k,l=1}^{\infty} x_{kl}$ is of a specified one of the above types, the transformed series $\sum_{k,l=1}^{\infty} x_{kl} b_{kl}$ will be of a specified one of these types. The process of transforming will be indicated by an arrow; "sufficient" will be abbreviated by *S.*, "necessary" by *N.* Thus $N.b \to c$ reads "a condition (or set of conditions) necessary that every bounded series have a convergent transform," and $S.b \to c$ reads "a condition (or set of conditions) sufficient that every bounded series have a convergent transform."

Hardy* found conditions *N.* and $S.rc \to rc$, and conditions $S.b \to rc$, and established relations (6) and (8) below. Kojima† proved the necessity of Hardy's conditions $S.b \to rc$, and discovered conditions *N.* and $S.c \to c$. C. N. Moore‡ established conditions *N.* and $S.bc \to bc$ incidentally, in proving a theorem

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* Hardy, *On the convergence of certain multiple series*, Proceedings of the Cambridge Philosophical Society, vol. 19 (1920), pp. 86–95. This paper will be referred to as H.

† Kojima, *Theorems on double series*, Tôhoku Mathematical Journal, vol. 17 (1920), pp. 213–220. This paper will be referred to as K.

‡ C. N. Moore, *On convergence factors in multiple series*, Transactions of this Society, vol. 29 (1927), pp. 227–238. Let $r=1$, and fix α and β in Moore's Theorem 1.