COMPLEXES AND MANIFOLDS REPRESENTED BY FUNCTIONS OF REAL VARIABLES*

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It is the purpose of this paper to show that a wide class of loci in a real space of n dimensions is composed of sets which are complexes and manifolds in the sense of combinatorial analysis situs. The combinatorial approach to analysis raises the important question of determining the conditions that a real function $u=f(x_1, \cdots, x_n)$ of a finite set of real variables must satisfy in order that this function generate a locus which can be identified as a cell, complex, or manifold.† Answers to this question form essential links between combinatorial analysis situs and real variable theory. Van der Waerden,‡ Lefschetz,§ Koopman and Brown,|| have studied this question for cases where the function u is analytic. S. S. Cairns¶ has investigated the cases that arise for functions u that are continuous together with their first partial derivatives. The present paper deals with functions u which are continuous but which are not required to have first partial derivatives at any points of their domains of definition. The restrictions placed on the functions are essentially those used by Hedrick and Westfall** in their general

* Presented to the Society, November 30, 1935.

† See Veblen, *Analysis Situs*, Colloquium Publications of this Society, vol. 5, part 2.

‡ Mathematische Annalen, vol. 102 (1929), pp. 360–361.

§ *Topology*, Colloquium Publications of this Society, vol. 12, chapter 8. On page 364, Lefschetz expressed a belief that van der Waerden (loc. cit.) was first to establish connections between functionally defined loci and complexes. It should be mentioned that a slightly earlier paper by W. M. Whyburn [this Bulletin, vol. 35 (1929), p. 706] contained a proof that a locus of a general type was a manifold and also carried a reference to results of this type which S. S. Cairns had obtained but had not published.

|| Transactions of this Society, vol. 34 (1932), pp. 231–251.

¶ See his papers in Annals of Mathematics, (2), vol. 35 (1934), pp. 579–587, and this Bulletin, vol. 41 (1935), pp. 549–552, where references to his earlier work will be found.

** Bulletin de la Société Mathématique, vol. 44, pp. 1–14, and Festschrift David Hilbert, Berlin, 1922, pp. 74–77, or Mathematische Annalen, vol. 85 (1922), pp. 74–77.