ON THE UNIVALENCY OF CESÀRO SUMS OF UNIVALENT FUNCTIONS*

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Introduction. Let

$$f(z) = z + \sum_{2}^{\infty} a_n z^n \tag{1}$$

be analytic and univalent for $|z| < 1$, that is, if z_1 and z_2 ($z_1 \neq z_2$) are any two points inside the unit circle, then

$$f(z_1) - f(z_2) \neq 0.$$

The partial sums $S_n(z)$ of $f(z)$,

$$S_n(z) = z + a_2 z^2 + a_3 z^3 + \cdots + a_n z^n, \tag{2}$$

in general are not univalent in the unit circle, though they are univalent for $|z| < 1/4$ as G. Szegö has shown.‡ In some cases, however, one can say that the partial sums are univalent in the whole unit circle. J. W. Alexander§ has shown, for example, that if the coefficients of (1) are real and positive and such that the numbers na_n form a decreasing sequence, then not only $f(z)$ but all its partial sums are univalent in the unit circle.

To the best of the writer's knowledge the only results obtained to date regarding the univalency of the Cesàro sums of univalent functions are those of L. Fejér|| who showed that if $f(z)$ is real on the real axis, and convex in the direction of the imaginary axis for $|z| < 1$, then all the Cesàro sums of the third order are univalent for $|z| < 1$.

In this paper we show that if the ordinary partial sums of

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‡ See G. Szegö, *Zur Theorie der schlichten Abbildungen*, Mathematische Annalen, vol. 100 (1928), pp. 188–211.

§ See J. W. Alexander, *Functions which map the interior of the unit circle upon simple regions*, Annals of Mathematics, (2), vol. 17 (1915–16), pp. 12–22.

|| See L. Fejér, *Neue Eigenschaften der Mittelwerte bei den Fourierreihen*, Journal of the London Mathematical Society, vol. 8 (1933), pp. 53–62.