

LINEAR DIFFERENTIAL EQUATIONS OF INFINITE ORDER*

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1. *Introduction.* In this address we shall give such a conspectus of the theory of linear differential equations of infinite order as will enable the reader rapidly to orient himself with respect to this subject. Neither in the presentation of results already developed nor in the bibliographical references (in the footnotes) is there any attempt to attain an exhaustive account. The purpose is rather that of a general outlook on the subject such as may interest a considerable number of persons and may serve as a point of departure for a few investigators who may desire to penetrate a relatively new and unexplored domain, the importance of which will certainly be more fully recognized as the subject is further developed in the next two or three decades.

In §2 the general nature of the problem of linear differential equations of infinite order is indicated partly by means of examples and partly by means of notions of a general character which are intimately associated with the somewhat more special problem which is the center of our present interest. In §3 are set forth some of the many connections of this problem with other matters of more or less wide interest in the field of analysis. A brief account of the present state of knowledge with reference to linear differential equations of infinite order with constant coefficients is given in §4, while §§5 and 6 treat (but with somewhat less fullness) the corresponding matters for equations with polynomial and with analytic coefficients, respectively. Finally, §7 is devoted to a brief account of some further problems and connections of the theory of differential equations of infinite order.

2. *Nature of the Problem.* A typical problem in the theory of linear differential equations of infinite order is that of solving the equation

$$(1) \quad a_0(x)y + a_1(x)y' + a_2(x)y'' + \cdots = \phi(x),$$

* Retiring address of the Chairman of Section A of the A.A.A.S., delivered at St. Louis, December 31, 1935.