POSTULATES FOR SPECIAL TYPES OF GROUPS

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In defining a special type of group, such as commutative group or finite group, one may be content to add a suitable postulate, or possibly a number of postulates, describing the special property under consideration, to any set of postulates for a general group. It is, however, of interest to pursue the matter further, to determine whether a simplified set of postulates can be set up which will adequately describe the special type. A number of investigations of this sort have been made; the results of Weber, Huntington, and Hurwitz are especially worthy of note.†

In the present paper I shall use as a basis the three-postulate definition of group which I recently presented in this Bulletin.‡ Let there be given a set of elements $G(a, b, c, \cdots)$ and a rule of combination, which may be called multiplication, by which any two elements, whether they be the same or different, taken in a specified order, determine a unique result which may or may not be an element of G. This system forms a group if it satisfies the following three postulates:

I. *If a, b, c, ab, bc, $(ab)c$, $a(bc)$ are all elements of G, then $(ab)c=a(bc)$.*

II. *If a and b are elements of G, there exists an element x of G such that $ax=b$.*

III. *If a and b are elements of G, there exists an element y of G such that $ya=b$.*

The reader will recall that a familiar four-postulate definition of group employs these three postulates, and a closure postulate. That definition was due to Huntington and Moore and was, in

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* As was reported in this Bulletin, vol. 41, p. 781, the author of this paper died on November 7, 1935. He had not seen the proofs of this paper. THE EDITORS.

† Weber, *Lehrbuch der Algebra*, vol. 2, 1896, pp. 3–4; Huntington, Transactions of this Society, vol. 4 (1903), pp. 27–29, and vol. 6 (1905), pp. 22–24 and p. 186; Hurwitz, Annals of Mathematics, (2), vol. 8 (1907), p. 94, and vol. 15 (1913), pp. 93–94.

‡ Vol. 40 (1934), pp. 698–701.