

ON CERTAIN VARIETIES WHOSE CURVE SECTIONS
ARE HYPERELLIPTIC CURVES*

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The properties of hyperelliptic curves, which have been described by Bobek,† are well known. One important property of such a curve, which must be of genus greater than unity, is that it carries on it one and only one complete and special series of groups of two points. From this property various others follow. For example, a hyperelliptic curve in any space can be transformed into a plane curve of order n with an $(n-2)$ -fold point. Now surfaces in a space of any number of dimensions whose sections by spaces of dimension one lower are hyperelliptic curves have certain properties and these have been investigated by Castelnuovo.‡ Such a surface must contain ∞^1 conics such that through each point of it passes one and only one of them. It can be transformed or projected into one, of order n , in a 3-dimensional space having an $(n-2)$ -fold line; and it is rational.

In this note we call attention to two varieties, in a higher space, whose curve sections are, as we shall show, hyperelliptic curves. One is the V_n^{2n+1} in S_{2n+1} which was the subject of an investigation by Babbage§ and the other is the $V_k^{2n-2k+1}$ in S_n which is the residual intersection of $(n-k)$ cubic hypersurfaces having in common the intersection M_{n-2}^4 of two quadric hypersurfaces of S_n . We shall describe in some detail the surface $V_k^{2n-2k+1}$ for $k=2$ or F^{2n-3} and also its projections in a 3-space. Incidentally, we shall obtain a property concerning linear spaces

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† Bobek, *Ueber hyperelliptische Curven*, *Mathematischen Annalen*, vol. 29 (1887), pp. 386–412.

‡ Castelnuovo, *Sulle superficie algebriche le cui sezioni piane sono curve iperellittiche*, *Rendiconti del Circolo Matematico di Palermo*, vol. 4 (1890), pp. 73–88.

§ Babbage, *A series of rational loci with one apparent double point*, *Proceedings of the Cambridge Philosophical Society*, vol. 27 (1931), pp. 399–403. See also B. C. Wong, *On a certain rational V_n^{2n+1} in S_{2n+1}* , *American Journal of Mathematics*, vol. 56 (1934), pp. 219–224.