A SET OF INDEPENDENT CONDITIONS THAT A REAL FUNCTION BE EVERYWHERE DIFFERENTIABLE†

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1. *Introduction.* The purpose of this paper is to give a set of conditions which characterize those real functions of a real variable which are differentiable‡ for all values of that variable. These conditions are somewhat analogous in spirit to the set of fundamental properties of definite integrals formulated by Lebesgue.§

In §2 we state the conditions, in §3 we discuss their relation to differentiation, and in §4 we prove that they are independent.

2. *The Conditions.* We consider a set C of real functions of the real variable x, $(-\infty < x < \infty)$. We suppose that to each function $f(x)$ in C there corresponds a real function $\bar{f}(x)$, $(-\infty < x < \infty)$, not necessarily in C. We impose on the set C and on the correspondence between $f(x)$ and $\bar{f}(x)$ certain restrictions which are formulated as four conditions:||

I. *The function $X(x) = x$ is a function in C; moreover, there exist constants x_1 and q such that $\overline{X}(x_1) = 1$ and $\overline{X}(x)$ is different from q for all x.*

† This paper is a revision of a communication presented to the Society, December 27, 1934, under the title *A set of completely independent postulates for differentiation.*

‡ We consider a function differentiable for $x = x_0$ if and only if the difference quotient approaches a finite limit for $x = x_0$.

§ H. Lebesgue, *Leçons sur l'Intégration et la Recherche des Fonctions Primitives* (*Borel Collection*), 1904, pp. 98, 99.

|| It will be noted that Condition I is compound in nature, inasmuch as it can be resolved into the following three propositions:

I′. *The function $X(x) = x$ is in C.*

I″. *If the function $X(x) = x$ is in C, then there exists a constant x_1 such that $\overline{X}(x_1) = 1$.*

I‴. *If the function $X(x) = x$ is in C, then there exists a constant q such that $\overline{X}(x)$ is different from q for all x.*

Condition III is also compound.