

exhibited by objects  $y_3$  to  $y_n$  if and only if  $x_1, x_2$ , and  $y_3$  to  $y_n$  exhibit  $\phi$ . By  $n-1$  such cumulative steps of interpretation, we find  $(\dots((\phi x_1)x_2)\dots)x_{n-1}$  to be the "monadic relation" or attribute of being an object  $y_n$  such that the objects  $x_1$  to  $x_{n-1}$  and  $y_n$  exhibit  $\phi$ . The whole proposition applies this attribute to  $x_n$  and thus tells us that the objects  $x_1$  to  $x_n$  exhibit  $\phi$ .

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## REMARK ON A RECENT PAPER BY HOLLCROFT

BY A. R. WILLIAMS

1. *Introduction.* Among the characteristics of the general web of quadric hypersurfaces in  $S_r$  described by T. R. Hollcroft in a recent paper\* is the number of lines on the jacobian surface of the web, that is, the number of hyperquadrics belonging to the web that have a line of vertices. This and more difficult questions are treated elegantly by associating the hyperquadrics of the web with the planes of a three-space. A direct algebraic treatment of the first mentioned problem may be of interest.

2. *Algebraic Formulation of the Problem. The Web of Conics.* For a quadric to have a line of vertices it is necessary and sufficient that all the first minors, but not all the second minors, of its discriminant vanish. This is three essential conditions. That is, in the linear system, or web,  $\lambda_1 f_1 + \lambda_2 f_2 + \lambda_3 f_3 + \lambda_4 f_4$ , where the  $f$ 's are linearly independent quadratic forms in any number of variables, a certain number have a line of vertices. While not strictly necessary it will make for clearness to begin with a web of conics. Let the discriminant be

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix},$$

where the elements are linear homogeneous functions of  $\lambda_1, \lambda_2, \lambda_3, \lambda_4$ , and  $a_{ij} = a_{ji}$ . The three first minors in the first two rows represent quadrics which have a cubic  $k$  in common. Any two of them

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\* This Bulletin, vol. 41 (1935), p. 97.