A REINTERPRETATION OF SCHÖNFINKEL'S LOGICAL OPERATORS

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The method in mathematical logic invented by Schönfinkel† and developed in detail by Curry‡ is important in that it completely eliminates the *variable* from the formal presuppositions of logic and mathematics. Constructed according to Schönfinkel's scheme the primitive language of mathematical logic consists only of a few constants; variables, if wanted as a convenience, are introduced afterward through conventions of shorthand.

Central to Schönfinkel's scheme is the device of construing relations as unitary operators. This is accomplished in the case of a dyadic relation ϕ by construing the proposition ϕxy (x bears the relation ϕ to y) as $(\phi x)y$, that is, as the proposition predicating of y an attribute ϕx which, in its turn, is the result of applying an operator ϕ to x. (The present recourse to variables is expository only, and foreign to the formal system.) Thus dyadic relations are for Schönfinkel unitary operators yielding attributes, where attributes may, for uniformity, be regarded in turn as unitary operators yielding propositions. In general, any n-adic relation is construed in corresponding fashion by taking the proposition $\phi x_1 x_2 \cdots x_n$ as $(\cdots((\phi x_1)x_2)\cdots)x_n$; n-adic relations become unitary operators yielding unitary operators $\cdots$ yielding unitary operators yielding propositions.

My present concern is to point out that Schönfinkel's explanation of relations as unitary operators is gratuitous: inessential to the net interpretation of his formulas, and avoidable by a slight reinterpretation of his notation. The new interpretation is advanced not necessarily as an improvement in an intuitive

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† Mathematische Annalen, vol. 92 (1924), pp. 305–316.

‡ American Journal of Mathematics, vol. 51 (1929), pp. 363–384; ibid., vol. 52 (1930), pp. 509–536, 789–834; ibid., vol. 54 (1932), pp. 551–558; Annals of Mathematics, (2), vol. 32 (1931), pp. 154–180; ibid., vol. 34 (1933), pp. 381–404; ibid., vol. 35 (1934), pp. 849–860; Proceedings of the National Academy of Sciences, vol. 20 (1934), pp. 584–590.