ON THE METRIC REPRESENTATIONS OF AFFINELY CONNECTED SPACES

BY T. Y. THOMAS

In a recent paper in the Transactions of this Society,* I defined the general concept of the algebraic characterization in complex differential geometry and proved the non-existence of a *simple* algebraic characterization of the metric spaces in the class of all complex affinely connected spaces. The following note is intended as an addendum to this paper which is to be consulted for the notations and definitions here used.

Assume that $F_1=0$, $F_2\neq 0$, where the F's represent polynomials in the components of the affine connection and their derivatives to an arbitrary but finite order, is necessary and sufficient for a complex affinely connected space to admit an n-dimensional metric representation. Then $B^{\alpha}_{\beta\gamma\delta}=0$ implies $F_2\neq 0$. Hence one of the polynomials P of the set F_2 must involve a non-vanishing constant term C so that we may put $P=Q+C$ with the understanding that each term of the polynomial Q depends on a component Γ or its derivative.

Consider the one parameter family of metric spaces S_a defined in a neighborhood U of the origin of the complex n-dimensional number space. For the affinely connected space S defined as the limit space of S_a as $a\rightarrow 0$, we have

$$\Gamma^1_{II} = -\frac{1}{2}\theta_1, \qquad (I = 2, \cdots, n),$$

$$\Gamma^1_{1\alpha} = \frac{1}{2}\theta_\alpha, \qquad (\alpha = 1, \cdots, n),$$

where

$$\theta = \log \phi, \qquad \theta_\alpha = \frac{\partial\theta}{\partial x^\alpha},$$

and ϕ is an analytic function of the coordinates $x^1, \cdots, x^n$ in the neighborhood U different from zero at $x^\alpha=0$; all other com-

* *Algebraic characterizations in complex differential geometry*, vol. 38 (1935), pp. 501–514.