to come, by bringing within their immediate reach the best of what has been achieved in the theory of Fourier series.

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ZARISKI ON ALGEBRAIC SURFACES

Algebraic Surfaces. By Oscar Zariski. Ergebnisse der Mathematischen Wissenschaften, Volume 3, Berlin, 1935. v+198 pp.

We are facing today, in the birational geometry of surfaces and varieties, more than in any other chapter of mathematics, the sharp need of a thoroughgoing and critical exposition. In a subject reaching out in so many directions, the task is bound to be arduous. Nevertheless it is surely urgent and for two reasons. In the first place, a systematic examination of the positions acquired is destined to be of considerable value in subsequent campaigns. In the second place, the territory already conquered and safely held is exceedingly beautiful and deserves to be admired by tourists and not merely by members of the vigorous, but small, conquering army. To speak less metaphorically, in this quarter of mathematics "cantorian" criticism has not penetrated as deeply as in others. This has resulted in a widespread attitude of doubt towards the science, which it would be in the interest of all to dispel as rapidly as possible. Nothing will contribute more to this worthy end than Zariski's splendid book. It is indeed the first time that a competent specialist, informed on all phases of the subject, has examined it carefully and critically. The result is a most interesting and valuable monograph for the general mathematician, which is, in addition, an indispensable and standard *vade mecum* for all students of these questions.

As is well known, when one endeavors to pass from one-dimensional birational geometry to the higher dimensions, the difficulties multiply enormously. Many results do not extend at all, or if they do, they are apt to assume a far more complicated aspect or else to demand most difficult proofs.

Consider for example these two questions: (a) the reduction of singularities; (b) the extension of the properties of the genus of an algebraic curve. For some time we have had quite complete and satisfactory proofs of the fact that any irreducible algebraic curve is birationally transformable into a non-singular curve in some space, or to a plane curve with simple and harmless singularities. A similar result is certain to hold for surfaces and varieties. Not to speak of varieties where complete obscurity still reigns, the proof for surfaces has given rise to much confusion. In Zariski's monograph we find the first critical and complete survey of the situation ever made. From this survey it appears, incidentally, that the only "certifiable" proof now in existence is R. J. Walker's (Annals of Mathematics, April, 1935).

A similar service has been performed by Zariski as regards the theory of the *irregularity* q, the analog of the genus p for a surface. The genus p of an algebraic curve is susceptible of four unrelated definitions: projective, birational-geometric, transcendental, topological. On passing to surfaces, these definitions give rise to different genera, which are distinct but not wholly independent.