ZYGMUND ON TRIGONOMETRIC SERIES

Trigonometric Series. By Antoni Zygmund. Warszaw, Monografje Matematyczne, Volume V, 1935. iv+320 pp.

It has been a repeated privilege of the reviewer to express his appreciation of the high standards and excellent quality of the series of monographs of which Volume V is now under his consideration. Each volume of the series published so far represents an important event in the development of mathematical research, and the present volume in this respect is second to none of its predecessors. If one looks through the long list of books on Fourier series one can not help feeling that even the bulkiest of them are far from giving an adequate picture of the present status of the field. The non-existence of a monograph giving such a picture was very badly felt not only by the beginners but also by specialists, and the failure of so many attempts to write a real book on Fourier series created an impression that the task was almost hopeless. The author of the present monograph completely succeeded in dispelling this "inferiority complex" and produced a book which not only introduces the reader into the immense field of the theory of Fourier series but at the same time almost imperceptibly brings him to the very latest achievements, many of them being due to the author himself. The style of the book is rigorous and vigorous and the exposition elegant and clear to the smallest details.

Without wasting his and the reader's time on unessential things the author endeavors to treat each special problem by methods throwing light on the problem from a general standpoint, and showing the place occupied by it in the whole structure. Such a method of exposition will prove to be extremely helpful to a neophyte and will delight a specialist.

Although Zygmund's monograph is far from being the bulkiest of all books written on Fourier series, it certainly contains the richest material. In fact there are but very few topics of importance omitted in it. Thus it is futile to attempt to present here an adequate idea of the subjects treated in the book, and we shall have to restrict ourselves to a very brief description by chapters. Chapters 1 (Trigonometrical series and Fourier series) and 2 (Fourier coefficients, tests for the convergence of Fourier series) are of an introductory character. However, even at this stage the author gives a rather complete discussion of various convergence tests and their mutual relationships, of the order of Fourier-Lebesgue and Fourier-Riemann coefficients, and of operations on Fourier series. Chapter 3 (Summability of Fourier series) contains a rapid but inclusive and elegant discussion of Cesàro and Abel summability of Fourier and Fourier-Stieltjes (derived) series and their conjugates. Chapter 4 (Classes of functions and Fourier series) deals with necessary and sufficient conditions which have to be satisfied by the Fourier series of a function in order that this function should belong to a certain specified function space (such as L_p, and a more general space L_ϕ, continuous, bounded and measurable, and the like). The generalized Parseval identity and the theory of factor sequences transforming one class of functions into another find their appropriate place here. A