LINEAR CONNECTIONS OF NORMAL SPACE TO A VARIETY IN EUCLIDEAN SPACE*

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1. *Introduction.* This paper deals with an extension of the Gauss formulas of a surface imbedded in ordinary space to apply to an m-dimensional variety imbedded in an n-dimensional euclidean space. The Gauss and Codazzi relations are extended to give integrability conditions in terms of the Christoffel symbols of the second kind of the variety, a set of tensors corresponding to the coefficients of the second fundamental form of a surface in three space and a set of non-covariant quantities: the connection coefficients of the normal space.

Several authors† have considered generalizations of the Frenet or the Gauss formulas to apply to varieties lying both in euclidean and in more general spaces, but the nature of some of the coefficients which may appear in these formulas seems to have escaped serious study.

In the considerations here there will be encountered an imbedding euclidean space of n dimensions, a variety of m dimensions lying in it and at each point of this variety an m-dimensional tangent space and an $(n-m)$-dimensional normal space. A vector in the imbedding space will be denoted simply by a letter, and all indices running from 1 to n will be suppressed. Latin indices lying between a and k will have the range from 1 to m and Latin letters from p through z when used as indices will have the range from 1 to $(n-m)$. Corresponding Greek letters will be used as summation indices.

The work in this paper differs from that in most of the previ-

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† See, for instance, Voss, Mathematische Annalen, vol. 16 (1880), p. 129; H. Weyl, Mathematische Zeitschrift, vol. 12 (1922), p. 162; Schouten and van Kampen, Mathematische Annalen, vol. 105 (1931), p. 144; E. Bortolotti, Rendiconti del R. Istituto Lombardo di Scienze e Lettere, (2), vol. 64 (1931), p. 441; E. H. Cutler, Transactions of this Society, vol. 33 (1931), p. 832; C. E. Weatherburn, Reports of the Australian and New Zealand Association for the Advancement of Science, vol. 21 (1933), p. 12; Duschek and Mayer, *Lehrbuch der Differentialgeometrie*, vol. 2.