A GENERALIZATION OF THE BERNOULLI POLYNOMIAL OF ORDER ONE*

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1. *Introduction.* The idea of a Bernoulli polynomial, generalized so as to allow of a continuous variation of the index of its degree, is not a new one.† In the present paper the writer has developed a simple definition of a generalized Bernoulli polynomial of order one which brings the Bernoulli polynomial into direct relationship with the generalized Riemann zeta-function. Another very interesting property of this generalization of the Bernoulli polynomial is brought out in §7.

2. *Difference Equation Considered.* For the purposes of this paper in dealing with a complex power s of a complex number t, if $t=re^{i\theta}$, then $t^s \equiv t^{\sigma+i\tau}=e^{s[\log r+i\theta]}$, $(-\pi<\theta\leqq\pi)$. The following *difference equation* will frequently be referred to:

$$(1) \qquad \frac{1}{w}\left[f(x+w,s)-f(x,s)\right]=sx^{s-1},$$

where the difference interval w is taken real and positive. Also there will be occasion to impose the *asymptotic condition: for any value of s such that $R(s)<0$,*

$$(2) \qquad f(x,s)\rightarrow 0 \quad as \quad R(x)\rightarrow +\infty.$$

There will also be occasion to refer to the following regions on the x and s planes:

Region X. All points on the x plane other than the negative axis of reals and the origin.

Region S. All points on the s plane within and on the boundary of a circle of radius M centering at the origin, (usually taken arbitrarily large, see §5).

3. *Uniqueness of an Analytic Solution.*

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† See N. E. Nörlund, *Vorlesungen über Differenzenrechnung*, p. 53. Future references to this book will be indicated by the letter N.