CORRESPONDENCES CONNECTED WITH A PENCIL OF n-ICS*

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1. *Introduction.* All curves of order n passing through $(n/2)(n+3)-1$ points pass through $(n-1)(n-2)/2$ other points. That is, the n^2 base points of a pencil of n-ics are determined by the former number. When $(n/2)(n+3)-2$ are fixed in general position, and another moves on a curve of order m, the locus of the remaining $(n-1)(n-2)/2$ is a curve of order $m(n^2-1)$, which has a multiple point of order mn at each of the fixed points.† In a previous paper I have considered the case when a number of base points are fixed and the others necessary to determine a pencil are taken consecutive on some curve.‡ One other situation is perhaps worth brief notice. Of the nq intersections of two curves of order n and q, $n>q$, $nq-(q-1)(q-2)/2$ can be taken arbitrarily, the others being then determined. Moreover,

$$nq - \frac{(q-1)(q-2)}{2} + \frac{(n-q)(n-q+3)}{2} - 1 = \frac{n(n+3)}{2} - 2.$$

This means that if $nq-(q-1)(q-2)/2$ base points are taken on a curve of order q, and $(1/2)(n-q)(n-q+3)-1$ others not on the latter, then if one other, say P, describes some locus, $(q-1)(q-2)/2$ will be fixed on the curve of order q, and the remaining $(1/2)(n-1)(n-2)-(1/2)(q-1)(q-2)$ $=(1/2)(n-q)(n+q-3)$, variable with P, will, for any position of P, lie on the curve of order $n-q$ determined by P and the second set of fixed points just mentioned. It is the purpose of this paper to discuss the locus of the variable points when P describes a given curve.

2. *Order of the Locus and Its Singularities at the Base Points.* In this connection we may use the rational surface whose plane

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† Milinowski, Journal für Mathematik, vol. 77, p. 263.

‡ Williams, this Bulletin, vol. 36, p. 133.