$$
(25)\qquad \begin{aligned} &E\{(x_1 - a_1)^k(x_2 - a_2)^l \cdots (x_r - a_r)^m\} \\ &= (p_1e^{D_1} + p_2e^{D_2} + \cdots + p_re^{D_r})^n \cdot x_1^k x_2^l \cdots x_r^m \Bigg]_{\substack{x_1=-a_1 \\ \vdots \\ x_r=-a_r}}, \end{aligned}
$$

where $D_1 = \partial/\partial x_1$.

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ON A RESULTANT CONNECTED WITH FERMAT'S LAST THEOREM

BY EMMA LEHMER

E. Wendt* seems to have been the first to introduce the resultant of $x^n = 1$ and $(x+1)^n = 1$ in connection with Fermat's Last Theorem. This resultant can be expressed by means of the following circulant of binomial coefficients

$$
\Delta_n = \begin{vmatrix} 1 & C_{n,1} & C_{n,2} & \cdots & C_{n,n-1} \\ C_{n,n-1} & 1 & C_{n,1} & \cdots & C_{n,n-2} \\ \cdot & \cdot & \cdot & \cdots & \cdot \\ C_{n,1} & C_{n,2} & C_{n,3} & \cdots & 1 \end{vmatrix}.
$$

In his book on Fermat's Last Theorem Bachmann† proved that if p is an odd prime and if Δ_{p-1} is not divisible by p^3, then Fermat's equation $x^p + y^p + z^p = 0$ has no solution (x, y, z) prime to p.

S. Lubelsky‡ proved in a recent paper, using the distribution of quadratic residues, that if $p \geqq 7$, Δ_{p-1} is not only divisible by p^3, but by p^8, thus annulling Bachmann's criterion except for $p = 3$ and $p = 5$.

We shall now show how, by a straightforward manipulation with the above determinant, one can prove much more.

THEOREM 1. *Δ_{p-1} is divisible by $p^{p-2}q_2$ for every prime p, where q_2 is the Fermat quotient $(2^{p-1} - 1)/p$.*

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* Journal für Mathematik, vol. 113 (1894), pp. 335–347.

† *Das Fermatproblem*, 1919, p. 59.

‡ Prace Matematyczno-Fizyczne, vol. 42 (1935), pp. 11–44.