ON THE BERNOULLI DISTRIBUTION*

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1. *Introduction.* The Bernoulli or binomial distribution, of central importance in the theory of mathematical probability and statistics, has been the subject of considerable study. The derivation of the moments, or recursion formulas for the moments, of this distribution, as usually presented, involve the use of moment-generating or characteristic functions, or the explicit form of the distribution itself.† In the following we will derive these moments in an elementary manner and extend the results to the Poisson exponential distribution, distributions of the Lexis and Poisson types, and the multinomial distribution.

2. *Preliminary Notions.* We need and use the following assumption, definitions, and theorem.

EMPIRICAL ASSUMPTION. If an event which can happen in two different ways be repeated a great number of times under the same essential conditions, the ratio of the number of times that it happens in one way, to the total number of trials, will approach a definite limit, as the latter number increases indefinitely.‡

DEFINITION. The limit described in the empirical assumption shall be called the probability that the event shall happen in the first way under those conditions.§ We shall express the fact

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* Presented to the Society, November 30, 1934.

† See, for example, A. Fisher, *The Mathematical Theory of Probabilities*, 2d ed., p. 104 ff.; H. L. Rietz, *Mathematical Statistics*, 1927, p. 26 ff.; V. Mises, *Wahrscheinlichkeitsrechnung*, 1931, pp. 131–133; Risser and Traynard, *Les Principes de la Statistique Mathêmatique*, 1933, pp. 39–40 and 320–321; V. Romanovsky, *Note on the moments of the binomial* $(q+p)^n$ *about its mean*, Biometrika, vol. 15 (1923), pp. 410–412; A. T. Craig, *Note on the moments of a Bernoulli distribution*, this Bulletin, vol. 40 (1934), pp. 262–264; A. R. Crathorne, *Moments de la binomiale par rapport à l'origine*, Comptes Rendus, vol. 198 (1934), p. 1202; A. A. Krisknasuami Aygangar, *Note on the recurrence formulae for the moments of the point binomial*, Biometrika, vol. 26 (1934), pp. 262–264.

‡ J. L. Coolidge, *An Introduction to Mathematical Probability*, 1925, p. 4.

§ J. L. Coolidge, op. cit., p. 4.