A NOTE ON LIPSCHITZ CLASSES

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This note consists in the application of some results of Hardy and Littlewood* on fractional integrals to a theorem of Paley and Zygmund† and gives a generalization of that theorem.

We consider only functions of the Fourier power series type. That is, $f(x)$ is periodic in 2π, integrable, and with a Fourier series of the form

$$f(x) \backsim \sum_{n=0}^{\infty} c_n e^{inx}, \qquad c_n = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)e^{-inx}dx.$$

In dealing with functions of the class Lip(α) or Lip(α, p), $\alpha \neq 1$, this restriction is a matter of convenience rather than one of necessity.‡

A function $f(x)$ is said to belong to the class Lip(α), where $0 \leqq \alpha \leqq 1$, in the interval $(-\pi, \pi)$, if

$$f(x + h) - f(x - h) = O(h^\alpha)$$

uniformly for $-\pi \leqq x-h < x+h \leqq \pi$, and to Lip$(\alpha, p)$, where $p \geqq 1$, $0 \leqq \alpha \leqq 1$, in $(-\pi, \pi)$, if $f(x) \epsilon L$, and

$$\int_{-\pi}^{\pi} | f(x + h) - f(x - h) |^p dx = O(h^{\alpha p}).$$

The functions $\phi_n(t)$, $(n=0, 1, 2, \cdots)$, are the Rademacher functions.§

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* Hardy and Littlewood, *Some properties of fractional integrals* I, Mathematische Zeitschrift, vol. 27 (1927–28), pp. 565–606. We will refer to this paper as (HL).

† Paley and Zygmund, *On some series of functions*, Proceedings Cambridge Philosophical Society, vol. 26 (1930), pp. 337–357. A. Zygmund, *Trigonometrical Series*, 1935, §5.61. We will refer to this book as (Z). It contains extensive bibliographical references.

‡ Hardy and Littlewood, *A convergence criterion for Fourier series*, Mathematische Zeitschrift, vol. 28 (1928), pp. 612–634, in particular, §2 and §3.5. See also (Z), §7.4.

§ For definitions and properties see (Z), §§1.32 and 5.5 to 5.61.