DISTRIBUTION OF MASS FOR AVERAGES OF NEWTONIAN POTENTIAL FUNCTIONS

BY J. M. THOMPSON

1. *Introduction.* It has been proved that the average of a potential function over a spherical volume and the average of a potential function over a spherical surface are themselves potential functions.* This paper is concerned with the determination of the distribution of mass for these two spherical averages; in addition, the distribution of mass for more general averages is obtained.

2. *Preliminary Theorems.* The problem is solved by means of a theorem on the change of the order of integration of an iterated Stieltjes integral. First it is necessary to state some preliminary theorems. We recall the following elementary theorem.

If $h(Q)$ is continuous in Q and $g(e)$ is a distribution of positive mass, bounded in total amount, and lying on a bounded set F (which may be taken as closed without loss of generality), then, for the integral over the whole of space, w,

$$(1) \qquad \left| \int_w h(Q)dg(e_Q) - \sum_w h(Q_i)g(e_i) \right| < \omega_\delta \alpha,$$

where the summation is extended over all the meshes of a lattice L_δ, of diameter $\leqq \delta$, Q_i is a point of the mesh e_i, ω_δ is the oscillation of $h(Q)$ on a subset of F of diameter $\leqq \delta$, and $\alpha \geqq g(F)$.

This theorem will be applied to the integral

$$\int_w h^N(M, Q)dg(e_Q, P),$$

where $h^N(M, Q)$ is continuous in M, Q, and $g(e, P)$ and F are bounded independently of P, so that ω_δ and α in (1) are independent of M, P.

THEOREM 1. *If $g(e, P)$ is a distribution of positive mass, bounded independently of P, on a set F bounded independently*

* G. C. Evans, *On potentials of positive mass*, Transactions of this Society, vol. 37 (1935), p. 250.