

ON APPROXIMATION BY POLYNOMIALS TO A  
FUNCTION ANALYTIC IN A SIMPLY  
CONNECTED REGION\*

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In a previous paper† the writer studied expansions in series of polynomials of a function  $f(z)$  analytic in a limited simply connected region  $G$  where  $f(z)$  is known either to be bounded in  $G$  or such that the double integral over  $G$  of the  $p$ th power ( $p > 0$ ) of the modulus of  $f(z)$  exists.‡ The present note contains an extension of each of the two theorems obtained in the earlier paper. The extended theorems now read as follows.

**THEOREM A.** *Let  $G$  be a limited simply connected region of the  $z$  plane. Then in order that corresponding to every function  $f(z)$  analytic and bounded in  $G$  there shall exist a sequence of polynomials  $\{p_n(z)\}$  which converge to  $f(z)$  in  $G$  as  $n \rightarrow \infty$  and at the same time such that*

$$(1) \quad \overline{\lim}_{n \rightarrow \infty} [ |p_n(z)|, z \text{ in } G ] \leq \overline{\text{bound}} [ |f(z)|, z \text{ in } G ],$$

*it is necessary and sufficient that the boundary of  $G$  be also the boundary of an infinite region.*

**THEOREM B.** *In the  $z$  plane let  $G$  be a limited simply connected region whose boundary is also the boundary of an infinite region. Let  $f(z)$  be analytic in  $G$  and such that*

$$(2) \quad \iint_G |f(z)|^p dS, \quad (p > 0),$$

*exists. Then there exists a sequence of polynomials  $\{p_n(z)\}$  such that*

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