A CERTAIN THREE-DIMENSIONAL CONTINUUM*

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In this note there is given an example of a bounded continuum in three-dimensional euclidean space such that there exists a point A of M and subcontinua G, K_1, and K_2 of $M-A$ satisfying the following conditions: (1) $M-K_i$, $(i=1, 2)$, is the sum of two mutually separated point sets each of which is connected; (2) each point of G is separated from A in M by either K_1 or K_2; however, (3) there does not exist a point set consisting of a finite number of connected subsets of M and separating A from G in M.

This example was obtained in 1928 while the author was a student at the University of Texas. It answers in the negative for three-dimensional continua a certain question concerning continua proposed to the author by R. L. Moore at that time. R. E. Basye† has recently answered Moore's question in the affirmative for plane continua. It is because of Basye's result that the example of this note is now published.

EXAMPLE. Using rectangular coordinates, consider the points $p_i=(1/i, 0, 0)$, $p_0=(0, 0, 0)$, $t_i=(1/i, 0, 1)$, $t_0=(0, 0, 1)$, $s_i=(1/i, -1, 0)$, $s_0=(0, -1, 0)$, and $A=(0, 1, 0)$, $(i=1, 2, \cdots)$. If P and Q are two points we shall denote by PQ the closed straight line segment from P to Q. The continuum M is now defined as $M=\sum[Ap_n+p_ns_n+p_nt_n]+s_1s_0+s_0t_0+t_0t_1$, where it is understood that the summation is with respect to n over the values $n=0, 1, 2, \cdots$. The subcontinua G, K_1, and K_2 are defined as follows: $G=s_1s_0+s_0t_0+t_0t_1$, $K_1=\sum p_nt_n+t_0t_1$, $K_2=\sum p_ns_n+s_0s_1$. It is seen that $M-K_1=U_1+V$, where $U_1=\sum[s_np_n-p_n]+[s_0t_0-t_0]+s_0s_1$ and $V=\sum[Ap_n-p_n]$. Similarly, $M-K_2=U_2+V$, where $U_2=\sum[t_np_n-p_n]+[s_0t_0-s_0]+t_0t_1$. Clearly U_i and V, $(i=1, 2)$, are mutually separated point sets each of which is connected. Moreover, each point of G belongs to either U_1 or U_2. However, $M-(G+A)=\sum B_n$, where $B_n=\sum[Ap_n-A]+\sum[p_nt_n-t_n]+\sum[p_ns_n-s_n]$. Since for $n\neq m$, $(n, m=0, 1, \cdots)$,

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† See this issue of this Bulletin, pp. 670–674.