SASULY ON STATISTICS

Trend Analysis of Statistics—Theory and Technique. By Max Sasuly. Washington, D.C., The Brookings Institution, 1934. xiii+421 pp.

The formal procedure for fitting polynomial curves by the method of least squares has been available, from the investigations of Legendre, Gauss, and Laplace, since the early part of the nineteenth century. However, its utility was, until lately, seriously limited by the excessive labor entailed in carrying through the numerical computations even for polynomial curves of relatively low degree. The most fruitful research in recent years has been directed to the development of an effective technique to simplify the numerical computation steps in cases in which the data are equally spaced and equally weighted. As a result of these investigations, the practical difficulties which were formerly encountered in these cases have been entirely removed.

The factors which have contributed mainly to the notable advance which has thus lately been achieved in facilitating the practical least squares fitting of polynomial curves are the use of the orthogonal form of polynomials in the fitting procedure, together with the employment of the Gregory-Newton expansions for these orthogonal polynomials. The advantages of the orthogonal representation had long been recognized and, in fact, the foundations of the theory of least squares orthogonal polynomial fitting had been laid by Tchebycheff in the middle of the last century. However, the practical application of his methods was very laborious. Then, in 1913, the practical least squares fitting of polynomial curves was greatly facilitated by the proposal of Sheppard to use the Gregory-Newton form of expansion for the equation of the polynomial curve to be fitted, that is, an expansion in a series of factorial terms, $1, x, x(x-1)/2!, x(x-1)(x-2)/3!, \cdots$, rather than in the usual form of a series of power terms, $1, x, x^2, x^3, \cdots$, owing to the fact that an explicit solution for the parameters of a polynomial of any degree could then be secured. Moreover, by using that form of expansion, the values of the parameters were obtained in terms of factorial moments of the data, which could readily be computed by a repeated summation procedure. The final stage in simplifying the practical least squares fitting of polynomial curves consisted in combining the advantages secured from the orthogonal representation with those resulting from the use of the factorial form of expansion of the polynomials.

In the book under review, the author has developed the theoretical basis of the underlying formulas for fitting polynomial curves by the method of least squares in much the same order as that which was actually followed in the course of the historical development, as briefly indicated above. That is, the formulas for least squares fitting of polynomial curves for a few of the lower degree polynomials in the familiar power series form are first derived. Up to the present time, however, no direct general solution for the values of the parameters of least squares power polynomials in terms of the power moments of the data appears to be available. Thus, attention is directed to the Gregory-Newton form of expansion, where a relatively simple, general solution in terms